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Time : 55 minutes
Spring 2017-18

MATHEMATICS 218
QUIZ I

NAME --Key--
ID# ------------------

Circle your section number :

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1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
8 M	11 M	2 M	12 M	3 M	4 M	8 F	11 M	11 F	2 T	3:30 T	5 T	1 M	3 M	4 M

PROBLEM GRADE

PART I

1 ------- / 12

2 ------- / 10

3 ------- / 16

4 ------- / 12

PART II

5	6	7	8	9	10	11	12
a	a	a	a	a	a	a	a
b	b	b	b	b	b	b	b
c	c	c	c	c	c	c	c
d	d	d	d	d	d	d	d
e	e	e	e	e	e	e	e

5-12 ------- / 32

PART III

13	14	15	16	17	18
T	T	T	T	T	T
F	F	F	F	F	F

13-18 ------- / 18

TOTAL ------- / 100

PART I. Answer each of the following problems in the space provided for each problem (Problem 1 to Problem 4).

1. Find the values of k for which the following system

$$\begin{aligned} x + 2y - kz &= 0 \\ -x + 3y + k^2 z &= k \\ -5y - 2z &= 1 \end{aligned}$$

has

a. a unique solution
b. no solution
c. infinitely many solutions.

[12 points]

$$\left[\begin{array}{ccc|c} 1 & 2 & -k & 0 \\ -1 & 3 & k^2 & k \\ 0 & -5 & -2 & 1 \end{array}\right]$$

$$\xrightarrow{R_1+R_2} \left[\begin{array}{ccc|c} 1 & 2 & -k & 0 \\ 0 & 5 & k^2-k & k \\ 0 & -5 & -2 & 1 \end{array}\right]$$

$$\xrightarrow{R_2+R_3} \left[\begin{array}{ccc|c} 1 & 2 & -k & 0 \\ 0 & 5 & k^2-k & k \\ 0 & 0 & k^2-k-2 & k+1 \end{array}\right]$$

a) Unique solution :

$k^2-k-2 \neq 0$

$(k-2)(k+1) \neq 0$

$\boxed{k \neq 2 \;, \; k \neq -1}$

b) no solution :

$k^2-k-2 = 0$ and $k+1 \neq 0$

$(k-2)(k+1) = 0$ and $k \neq 1$

So $\boxed{k = 2}$

c) infinitely many solutions

$k^2-k-2 = 0$ and $k+1 = 0$

$k = 2, k = -1$ & $k = -1$

$\Rightarrow \boxed{k = -1}$

2. Let A be a 3x3 matrix such that $(A^{-1}-2I)^{-1}=\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix}$. Find A.

[10 points]

$$\left[\underbrace{\begin{matrix}0&0&1\\0&1&0\\1&0&0\end{matrix}}_{(A^{-1}-2I)^{-1}}\middle|\underbrace{\begin{matrix}1&0&0\\0&1&0\\0&0&1\end{matrix}}_{I}\right]\xrightarrow{R_1\leftrightarrow R_3}\left[\underbrace{\begin{matrix}1&0&0\\0&1&0\\0&0&1\end{matrix}}_{I}\middle|\begin{matrix}0&0&1\\0&1&0\\1&0&0\end{matrix}\right]$$

$$\text{So } A^{-1}-2I=\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix}$$

$$\text{So. } A^{-1}=\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix}+\begin{pmatrix}2&0&0\\0&2&0\\0&0&2\end{pmatrix}=\begin{pmatrix}2&0&1\\0&3&0\\1&0&2\end{pmatrix}$$

$$\Rightarrow A=(A^{-1})^{-1}$$

$$\left[\underbrace{\begin{matrix}2&0&1\\0&3&0\\1&0&2\end{matrix}}_{A^{-1}}\middle|\underbrace{\begin{matrix}1&0&0\\0&1&0\\0&0&1\end{matrix}}_{I}\right]\xrightarrow[\frac{1}{3}R_2]{R_1\leftrightarrow R_3}\left[\begin{matrix}1&0&2\\0&1&0\\2&0&1\end{matrix}\middle|\begin{matrix}0&0&1\\0&\frac{1}{3}&0\\1&0&0\end{matrix}\right]$$

$$\xrightarrow{-2R_1+R_3}\left[\begin{matrix}1&0&2\\0&1&0\\0&0&-3\end{matrix}\middle|\begin{matrix}0&0&1\\0&\frac{1}{3}&0\\1&0&-2\end{matrix}\right]$$

$$\xrightarrow{-\frac{1}{3}R_3}\left[\begin{matrix}1&0&2\\0&1&0\\0&0&1\end{matrix}\middle|\begin{matrix}0&0&1\\0&1/3&0\\-1/3&0&\frac{2}{3}\end{matrix}\right]$$

$$\xrightarrow{-2R_3+R_1}\left[\underbrace{\begin{matrix}1&0&0\\0&1&0\\0&0&1\end{matrix}}_{I}\middle|\underbrace{\begin{matrix}2/3&0&-1/3\\0&\frac{1}{3}&0\\-1/3&0&2/3\end{matrix}}_{A}\right]$$

$$\text{So } A=\begin{bmatrix}2/3&0&-1/3\\0&1/3&0\\-1/3&0&2/3\end{bmatrix}$$

$$A=\frac{1}{3}\begin{pmatrix}2&0&-1\\0&1&0\\-1&0&2\end{pmatrix}$$

3. Consider the matrix

$$A=\begin{pmatrix} 1 & 1 & 2 \\ -2 & -1 & -3 \\ 1 & 2 & 3 \end{pmatrix},$$

(a) Show that A is not invertible. [3 points]

Col.(1) + Col(2) = col(3)

$\Rightarrow |A| = 0$

$\Rightarrow$ A is not invertible

or show det(A) = 0

(b) For a real number a, consider the vector $\mathbf{b}=\begin{pmatrix} 1 \\ a-2 \\ a^2+a \end{pmatrix}$. Find the values of a such that the linear system AX=b (with $X=\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$) is consistent. [7 points]

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ -2 & -1 & -3 & a-2 \\ 1 & 2 & 3 & a^2+a \end{array}\right] \xrightarrow[-R_1+R_3]{2R_1+R_2} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & 1 & a \\ 0 & 1 & 1 & a^2+a-1 \end{array}\right]$$

$$\xrightarrow{-R_2+R_3} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & 1 & a \\ 0 & 0 & 0 & a^2-1 \end{array}\right]$$

Consistent $\Leftrightarrow a^2-1=0$

$\Leftrightarrow a=\pm 1$

(c) Let a=1 in (b). Find all the solutions $X=\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ of the linear system AX=b. [6 points]

$a=1 \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$\left.\begin{array}{l} x_1+x_2+2x_3=1 \\ x_2+x_3=1 \end{array}\right\} \Rightarrow x_3=t$ = free variable

$\Rightarrow x_2 = 1-x_3 = 1-t$

$\Rightarrow x_1 = 1-x_2-2x_3 = 1-1+t-2t = -t$

So Solution set $= \left\{ \begin{pmatrix} -t \\ 1-t \\ t \end{pmatrix} \middle| \, t \in R \right\}$ infinitely many solutions

4. (a) Let B be a 3×3 skew-symmetric matrix ($B^T = -B$). Prove that det (B)=0

[6 points]

$B^T = -B \Rightarrow |B^T| = |-B| = (-1)^3 \cdot |B|$ 3x3

$\Rightarrow |B| = -|B|$

$\Rightarrow 2 \det B = 0$

$\Rightarrow \det B = 0$

(b) Let A be a 2×2 matrix such that $A^2+A-I=\mathbf{0}$. Find A^{-1} and prove that A cannot be skew-symmetric.

[6 points]

$A^2 + A = I$

$\Rightarrow A.(A+I) = I$ So A is invertible and

$\Rightarrow A^{-1} = A + I$

Suppose A is skew symmetric 2x2 matrix

$\Rightarrow A = \begin{pmatrix} 0 & a \\ -a & 0 \end{pmatrix}$

$\Rightarrow A^{-1} = \frac{1}{|A|} \cdot \begin{pmatrix} 0 & -a \\ a & 0 \end{pmatrix} = \frac{1}{a^2} \cdot \begin{pmatrix} 0 & -a \\ a & 0 \end{pmatrix} = \begin{pmatrix} 0 & -\frac{1}{a} \\ \frac{1}{a} & 0 \end{pmatrix}$

But $A^{-1} = A + I = \begin{pmatrix} 0 & a \\ -a & 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & a \\ -a & 1 \end{pmatrix}$

So $\begin{pmatrix} 1 & a \\ -a & 1 \end{pmatrix} = \begin{pmatrix} 0 & -\frac{1}{a} \\ \frac{1}{a} & 0 \end{pmatrix} \Rightarrow 1 = 0$ impossible

So A cannot be skew symmetric

PART II. Circle the correct answer for each of the following multiple choice problems (Problem 5 to Problem 12) IN THE TABLE IN THE FRONT PAGE . [4 points for each correct answer, NO PENALTY FOR A WRONG ANSWER IN THIS PART].

5. Let A be a 3×3 skew-symmetric ($A^T = -A$) matrix. Which one of the following statements is **FALSE:**

(a) A^T is skew symmetric
(b) $(A^T - A)$ is skew symmetric
(c) $(A^2 - A)$ is symmetric
(d) $a_{11} = a_{22} = a_{33} = 0$
(e) $A^T A$ is symmetric

$$(A^2-A)^t = (A^2)^t - A^t = (A^t)^2 - A^t = (-A)^2 + A = A^2 + A$$

[4 points]

6. The value(s) of k for the system with corresponding augmented matrix $\left(\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 1 & k \end{array}\right)$ to be consistent are

$$\xrightarrow{-R_1+R_2} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & 1 & 1 & k \end{array}\right] \xrightarrow{-R_2+R_3} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & k+1 \end{array}\right] \quad k+1=0$$

(a) k=1
(b) k=−1
(c) k≠1
(d) k≠−1
(e) none of the above

[4 points]

7. Let A be an nxn symmetric matrix. Which one of the following statements is **FALSE**

(a) A^T is symmetric
(b) $3AA^T+A^2+2I$ is symmetric.
(c) A is invertible
(d) $A-A^T$ is symmetric

$A = \begin{pmatrix} 0 & \cdots & 0 \\ \vdots & & \vdots \\ 0 & \cdots & 0 \end{pmatrix}$ is symmetric but A is not invertible

[4 points]

8. Let A be a square matrix such that $A^3=A$. Then

(a) A is not invertible
(b) $\det(A)=\pm 1$
(c) If A is invertible then $A^{-1}=A$
(d) $A^7=A^2$
(e) None of the above

If A invertible $A^{-1}(A^3 = A)$
$\Rightarrow A^2 = I$
$\Rightarrow A^{-1} = A$

[4 points]

9. Let A be a matrix of the form

$$A=\begin{pmatrix} 1 & 0 & 1 & 4 \\ 0 & a & 0 & 2 \\ 0 & b & a & 0 \end{pmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & 4 \\ 0 & a & 0 & 2 \\ 0 & b & a & 2 \end{bmatrix}$$

$b \neq 0 \Rightarrow$ not echelon form

Which **one** of the following statements is **TRUE**?

a. If a=b=0, then A is not in row echelon form.
b. If a=1and b=0, then A is in reduced row echelon form.
c. If b is not 0, then A is not in row echelon form.
d. If a=0 and b=1, then A is in row echelon form.
e. none of the above

[4 points]

10. Let A be a 3x3 matrix such that det(A)=2. Which **one** of the following statements is **TRUE**?

a. det(A-2I)=0.
b. $\det(2A^t) = \det(A^4)$.
c. A^tA is not invertible.
d. $\det(2A) = \det(A^2)$.
e. none of the above

$|2A^t| = 2^3 \cdot |A^t| = 8|A| = 16$
$|A^4| = |A|^4 = 2^4 = 16$

[4 points]

11. Let A be an nxn matrix. Which one of the following statements is **FALSE**:

a. If AX=0 has only the trivial solution, then $A^2X=0$ has only the trivial solution.
b. If $A\neq 0$, then the matrix equation AX=**b** has a unique solution for all **b**.
c. If AX=**b** has infinitely many solutions for some **b** then A is not invertible.
d. If det(A) is not 0 then the reduced row echelon form of A is I.

[4 points]

12. Let $v_1=\begin{pmatrix}1\\2\\1\end{pmatrix}$, $v_2=\begin{pmatrix}1\\1\\0\end{pmatrix}$ and $v_3=\begin{pmatrix}3\\5\\2\end{pmatrix}$. Which one of the following statements is

FALSE:

(a) If $v_3 - v_2 = cv_1$, then c=2.
(b) $2v_3 - 4v_1 = 2v_2$
(c) The only values of c,d such that $cv_1 + dv_2 = v_3$ are c=2, d=1.
(d) There exists a real number c such that $v_3 - cv_2 = 0$.

[4 points]

$\begin{pmatrix}3\\5\\2\end{pmatrix} \neq c\begin{pmatrix}1\\1\\0\end{pmatrix}$

PART III. Answer **TRUE** or **FALSE** only, **IN THE TABLE IN THE FRONT PAGE** (**3 points for each correct answer, and -1 point penalty for each wrong answer**)

F 13. If A and B are symmetric $n \times n$ matrices, then AB is symmetric. $(AB)^t = B^t A^t = BA$

T 14. If A and B are square $n \times n$ matrices such that B is not invertible, then AB is not invertible. $|B| = 0 \Rightarrow |AB| = |A| . 0 = 0$

F 15. If A is a non-invertible 3×3 matrix, then the system $AX=\mathbf{b}$ is inconsistent

F 16. Every triangular 3×3 matrix is invertible. $\begin{pmatrix} 0 & 1 & 2 \\ 0 & 3 & 5 \\ 0 & 0 & 1 \end{pmatrix}$ is not invertible

T 17. If A and B are $n \times n$ matrices such that A is invertible and $(A^{-1}B)^t$ is invertible, then B is invertible.
$0 \neq |(A^{-1}B)^t| = |A^{-1}B| = |A^{-1}| . |B| = \frac{|B|}{|A|}$
$\Rightarrow |B| \neq 0$

T 18. If A is a 3×3 matrix such that $\det(2A^{-1}) = 4$, then $\det(A^t) = 2$

$4 = \det(2A^{-1}) = 2^3 . \det(A^{-1}) = \frac{8}{\det A}$

[18 points]

$\Rightarrow \det(A) = \frac{8}{4} = 2$

$\Rightarrow \det(A^t) = 2$